**Black-Scholes Option Pricing Model**

**1,It^o's Lemma**

**Suppose：**

:Price of a particular asset.

:Time.

:Return on the asset.

:Drift,the average rate of growth of the asset price, A predictable,deterministic component.

:*volatility,* the standard deviation of the returns.

:A random variable having a normal distribution with mean 0 and variance .



the stochastic differential equation:



:Fuction by Price of a particular asset　and Time.

**Proof:**

**With:**

Taylor's Theorem



**With:**





**With:**







**Then:**



**2,The Black-Scholes PDE**

**Suppose：**

:the value of an option, for a call and  for a put.

:the interest rate.

: Portfolio, containing one option and  units of the underlying stock.



**Proof:**

**With:**

It^o's Lemma



**With:**





**With:**

Invested  in riskless assets with interest rate .





**Then:**



**3,Solve The Black-Scholes PDE**

**Suppose：**







**Solve:**

**With:**

Black-scholes PDE;







**With:**

;

;



**With:**

;





**With:**







**With:**

Fourier transform

Gaussian integral















**Then:**



**4,Application To European Options**

**Suppose：**

**,**European options

 for a call option

for a put option

**Proof:**







**With:**



****

**With:**





**With:**

cumulative standard normal distribution function.

****







**Then:**



**Reference**

Taylor's Theorem



Fourier transform









**With:**









Gaussian integral

